

S342 June
S.S.C.E. 1993
FURTHER
MATHEMATICS 2
2 hours

2

WEST AFRICAN EXAMINATIONS COUNCIL

NIGERIA

Senior School Certificate Examination

June 1993

FURTHER MATHEMATICS 2

2 hours

Answer twelve questions in all: all the questions in Section A and four questions from Section B, with at least one question from each part.

In each question all necessary details of working, including rough work, must be shown with the answer.

Give answers as accurately as data and tables allow.

The following are provided for use in the examination:

- (a) *graph paper;*
- (b) *drawing paper for construction work.*

Turn over

SECTION A

[48 marks]

*Answer all the questions in this section.**All questions carry equal marks.*

1. Resolve $\frac{2x^2 - 1}{(x^2 - 1)(x - 2)}$ into partial fractions.
2. A circle touches the x axis at the point $P(-3, 0)$ and its centre C lies on the line $2x + y + 1 = 0$. Find the:
 - (a) coordinates of C ;
 - (b) radius of the circle;
 - (c) equation of the circle.
3. A school has 37 vacancies for teachers, out of which 22 are for English Language, 20 for History and 17 for Fine Art. Of these vacancies 11 are for both English Language and History, 8 for both History and Fine Art and 7 for English Language and Fine Art. Using a Venn diagram, find the number of teachers who must be able to teach:
 - (a) all the three subjects;
 - (b) Fine Art only;
 - (c) English Language and History, but not Fine Art.
4. The distance s metres after time t seconds covered by a particle moving along a straight line is given by
$$s = t^3 - t^2 - t - 2.$$
Find the:
 - (a) average acceleration between $t = 2$ and $t = 4$;
 - (b) acceleration when the velocity is zero.
5.
 - (a) Two boys and four girls are to be seated in a row. If the two boys are to sit together, find the number of ways in which this can be done.
 - (b) A box contains 12 identical marbles, 3 red, 2 blue, 2 green and 5 yellow. If a marble is drawn at random from the box, what is the probability that it is **either** yellow or red?

6. The distribution of marks scored in Statistics and Mathematics by ten students is given in the table below.

Mathematics (x)	11	20	23	42	48	50	57	64	80	90
Statistics (y)	26	23	35	46	44	50	50	58	68	70

- (a) Plot a scatter diagram of the distribution.
 (b) Draw an eye-fitted line of best fit.
 (c) Use your diagram to estimate a student's mark in Statistics if his mark in Mathematics is 40.
7. If $u = -2i + 5j$ and $v = i + j$,
 find, to the nearest degree, the angle between u and v .
8. If $F_1 = (10N, 045^\circ)$ and $F_2 = (6N, 120^\circ)$ are two forces, find the,
 (i) magnitude and
 (ii) direction,
 of their resultant.

SECTION B

[52 marks]

Answer four questions only from this section, with at least one from each part.

All questions carry equal marks.

PART I

PURE MATHEMATICS

9. (a) Given that

$$\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5} - 1} = p + q\sqrt{r},$$

find the values of the constants p , q and r .

- (b) Solve the equation:

(i) $\log_2 (x^2 - 2) = \log_2 (x - 1) + 1;$

(ii) $4^{x+1} - 9(2)^x = -2.$

Turn over

10. (a) Evaluate:

$$(i) \int_2^3 \frac{(x+1)}{(x-1)} dx,$$

correct to 3 decimal places;

$$(ii) \int \frac{dx}{x^2(1-4x)}$$

(b) Using the substitution $(1+x^2) = u$, or otherwise, evaluate

$$\int \frac{xdx}{(1+x^2)^{3/2}}$$

11. (a) Show that the determinant

$$\begin{vmatrix} x+y & x & x \\ x & x+y & x \\ x & x & x+y \end{vmatrix} = (3x+y)y^2$$

(b) Evaluate

$$\begin{vmatrix} 3 & 4 & 2 \\ 1 & -5 & 3 \\ 2 & 3 & 1 \end{vmatrix} \quad \text{Hence, solve}$$

the following equations:

$$3x + 4y + 2z = 4$$

$$x - 5y + 3z = -1$$

$$2x + 3y + z = 3$$

PART II

STATISTICS AND PROBABILITY

12. (a) The faces of a fair die are numbered 1, 2, 3, 4, 5, 6. If the die is thrown twice, what is the probability of obtaining a total score of 6?
- (b) The probabilities that two athletes P and Q will win a gold medal in a competition are 0.75 and 0.60 respectively. What is the probability that in the competition,
- (i) both P and Q will win gold medals,
 - (ii) neither of them will win a gold medal,
 - (iii) at least one of them will win a gold medal?
13. The ages, in years, of workers in a certain factory are known to be normally distributed with mean 50 and standard deviation 10.
- (a) If a worker is selected at random from the factory, what is the probability that the worker's age is
- (i) at least 30 years,
 - (ii) between 40 and 60 years?
- (b) If there are 1,000 workers in the factory, find the number of workers whose ages will be at most 75 years.

PART III

VECTORS AND MECHANICS

14. A block of mass 5 kg, placed on a plane inclined at an angle of 30° to the horizontal, slides down at a slow steady speed.
- (a) Find the:
- (i) frictional force between the block and the plane;
 - (ii) coefficient of friction.
- (b) If the angle of inclination of the plane had been $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$, find what would have been the acceleration of the block.
[Take $g = 10\text{ms}^{-2}$]

Turn over

15. A particle is projected from a fixed point O with a velocity of $u \text{ ms}^{-1}$ at an angle α to the horizontal.

(a) If the particle is at a point $P(x, y)$ after time t seconds, show that

(i) $x = ut \cos \alpha,$

(ii) $y = ut \sin \alpha - \frac{1}{2}gt^2,$

(iii) the equation of the path of the particle is

$$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha.$$

where g is the acceleration due to gravity.

- (b) Find the horizontal range of the particle and hence the maximum range.
(c) Find u given that $x = 300 \text{ m}$ and $y = 100 \text{ m}$, when $t = 10$ seconds.

(Take $g = 10 \text{ ms}^{-2}$).