

P4012 Nov.
WASSCE 2011
FURTHER
MATHEMATICS/
MATHEMATICS
(ELECTIVE) 2

$2\frac{1}{2}$ hours

2

Name:.....

Index Number:.....

THE WEST AFRICAN EXAMINATIONS COUNCIL

West African Senior School Certificate Examination

FURTHER MATHEMATICS/MATHEMATICS (ELECTIVE) 2

November 2011

[100 marks]

$2\frac{1}{2}$ hours

Write your **name** and **index number** in the spaces provided at the top right-hand corner of this booklet.

Answer **twelve** questions in all. All the **eight** questions in Section A and **four** questions from Section B, with at least **one** question from **each** part.

In **each** question, all necessary details of working, including rough work, **must** be shown with the answer.

Give answers as accurately as data and tables allow.

The use of non-programmable, silent and cordless calculator is allowed.

The following are provided for your use in the examination:

- (a) graph paper;
- (b) drawing paper for construction work.

SECTION A

[48 marks]

*Answer all the questions in this section.**All questions carry equal marks.*

1. Find the truth set of $\sin \theta + \cos 2\theta = 0$, $0^\circ \leq \theta \leq 360^\circ$.
2. Find the equation of the line which passes through the point $(3, -2)$ and is perpendicular to the line $3x + 2y - 4 = 0$.
3. Solve for x and y in the equations:
$$\log(x - 1) + 2\log y = 2\log 3,$$
$$\log x + \log y = \log 6.$$
4. Find the third term of the exponential sequence (G.P.)
 $(\sqrt{2} - 1), (3 - 2\sqrt{2}), \dots$
5. X and Y are two events such that $P(X \cup Y) = \frac{11}{15}$ and $P(X) = \frac{1}{3}$.
Find $P(Y)$ if events X and Y are
 - (a) mutually exclusive;
 - (b) independent.
6. (a) In a Physics examination, the mean mark of the first twelve students in a class is 60, that of the next twenty students is 50 and that of the remaining y students is x . What is the mean mark for the whole class in the examination, in terms of x and y ?

(b) A box contains 4 red and 3 blue identical balls. If two balls are picked at random, one after the other **without** replacement, find the probability that one is red and the other is blue.

7. Given that $\mathbf{n} = \begin{pmatrix} -12 \\ 5 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, find the vector \mathbf{q} such that $|\mathbf{q}| = 35$ and \mathbf{q} is in the direction of $(\mathbf{n} + 5\mathbf{s})$.
8. (a) A car moving on a straight road with constant acceleration has a velocity of 20 kmh^{-1} at one instant. If at 15 minutes later, it had a velocity of 50 kmh^{-1} , find the acceleration of the car.
- (b) A particle is projected vertically upwards with a speed of 40 ms^{-1} from a point on the ground. Find the maximum height reached.
[Take $g = 10 \text{ ms}^{-2}$]

SECTION B

[52 marks]

*Answer **four** questions **only** from this section, with at least **one** question from **each** part.*

All questions carry equal marks.

PART I

PURE MATHEMATICS

9. (a) Use the trapezium rule with ordinates at $x = 1, 2, 3, 4$ and 5 to calculate, correct to two decimal places, an approximate value for $\int_1^5 (2x + 8x^{-2}) dx$.
- (b) Given that $f: x \longrightarrow x^2 + 1$ and $g: x \longrightarrow \frac{3x + 1}{x - 1} \text{ } x \neq 1$, find :
- (i) $g \circ f$;
- (ii) $g \circ f(2)$.

10. (a) The gradient of a curve is given by $2x - 3x^2$. Find the equation of the curve if a point (1, 2) lies on it.
- (b) (i) Find the equations of the normals to the curve $y = x^2 - 1$ at the points where it cuts the x -axis.
- (ii) Find the coordinates of the point of intersection of the normals in (b)(i).
11. (a) Solve for x , y and z in the equations:
- $$\begin{aligned} 3x + 5y - 4z &= -5 \\ 6x + 3y - 5z &= 26 \\ -2x + 2y + z &= -11. \end{aligned}$$
- (b) A function g is defined by
- $$g(x) = \frac{3 - 4x}{x^2 + x - 6}.$$
- Express $g(x)$ in partial fractions.
12. Given the curve $y = x^2 - 4$, calculate, correct to two decimal places, the:
- (a) area of the finite region bounded by the curve and the x -axis;
- (b) volume generated by rotating the region in (a) through 360° about the x -axis. [Take $\pi = \frac{22}{7}$].

PART II
STATISTICS AND PROBABILITY

13. (a) A committee of **five** is to be formed among 6 Ghanaians, 8 Nigerians and 5 Gambians. In how many ways can the committee be formed if;
- (i) there is no restriction;
 - (ii) at **most** 2 Ghanaians are on the committee;
 - (iii) 1 Nigerian is on the committee?
- (b) **Five** out of 12 articles are known to be defective. If **three** articles are picked, one after the other without replacement, find the probability that all the three articles are non-defective.
14. The number of cars that called at a petrol station on some days of a month is as shown in the table.

Day of the month (x)	3	5	8	12	15	19	22	26
Number of cars (y)	143	95	112	110	104	86	78	69

- (a) Represent this information on a scatter diagram.
- (b) Draw the *line of best fit* to pass through the point (\bar{x}, \bar{y}) , where \bar{x} is the mean of x and \bar{y} is the mean of y .
- (c) Use your diagram to estimate
 - (i) the day 80 cars called at the station;
 - (ii) how many cars a petrol attendant at the station should expect on the 25th day.

15. (a) The probability that a patient recovers from a disease is 0.25. If 6 people are known to have contracted this disease, calculate the probability that:
- (i) more than three people survived;
 - (ii) at **most** 2 people survived.
- (b) Two distinct numbers are selected at random from the set $P = \{ 2, 3, 4, 5, 6 \}$. Find the probability that
- (i) the sum of the two numbers is 8;
 - (ii) one of the numbers is a factor of the other.

PART III

VECTORS AND MECHANICS

16. (a) The position vectors of points P , Q and R are $5\mathbf{i} + 3\mathbf{j}$, $8\mathbf{i} - \mathbf{j}$ and $11\mathbf{i} - 5\mathbf{j}$ respectively.
- (i) Show that P , Q and R are collinear.
 - (ii) Find the scalars k_1 and k_2 such that $37\mathbf{i} - \mathbf{j} = k_1\mathbf{p} + k_2\mathbf{r}$ where \mathbf{p} and \mathbf{r} are position vectors of P and R respectively.
- (b) Given that $\mathbf{m} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{n} = 6\mathbf{i} + 4\mathbf{j}$, find the angle between the two vectors, correct to the nearest degree.

17. (a) A particle is projected vertically upwards with a speed of 25 ms^{-1} from a point on the ground. Find the:
- (i) position of the particle after 4 seconds;
 - (ii) maximum height reached;
 - (iii) time taken to reach the maximum height;
 - (iv) time when the particle is 30 m above the ground.

[Take $g = 10 \text{ ms}^{-2}$]

- (b) Calculate the force which acts on a body of mass 3 kg moving at 2.5 ms^{-1} for 0.5 seconds, if the final velocity is 4.5 ms^{-1} .

18. (a) A uniform bar PQ of length 60 cm and weight 20 N is supported at two points C and D such that $|PC| = 10 \text{ cm}$ and $|QD| = 15 \text{ cm}$. Two forces 10 N and 15 N are placed at P and Q respectively. If the system remains in equilibrium under the action of these forces, calculate the reactions at C and D .

[Take $g = 10 \text{ ms}^{-2}$]

- (b) Two forces $3\mathbf{i} \text{ N}$ and $4\mathbf{j} \text{ N}$ act on an object of mass 5 kg . Find the acceleration of the object.